**LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034**

**B.Sc. DEGREE EXAMINATION – MATHEMATICS**

**FOURTH SEMESTER – NOVEMBER 2012**

**MT 4502/4500 – MODERN ALGEBRA**

**Date: 03-11-2012 Dept. No. Max. : 100 Marks**

**Time: 1.00 – 4.00**

 **SECTION – A (10 × 2 = 20)**

**Answer ALL the questions.**

1. Define partially ordered set and give an example.
2. Show that identity element of a group is unique.
3. Let $a\in G$and $a^{n}=e$. Prove that $O(a)$divides $n$.
4. Show that any group of order up to 5 is abelian.
5. Define kernel of a homomorphism.
6. State fundamental theorem of homomorphism.
7. Define a ring and give an example.
8. If $R$ is a ring with unit element $1$, then for all $a\in R$, show that $\left(-1\right)a=a\left(-1\right)=-a$.
9. State unique factorization theorem.
10. If $R$ is a commutative ring with unity, prove that every maximum ideal of $R$ is a prime ideal.

**PART – B ( 5 × 8 = 40)**

**Answer any FIVE questions**

1. If $G$ is a group in which $\left(ab\right)^{k}=a^{k}b^{k}$ for three consecutive integers $k$ for all $a,b,c\in G$, show that $G$ is abelian.
2. Show that a subgroup $N$ of a group $G$ is a normal subgroup of $G$ if and only if every left coset of $N$ in $G$ is a right coset of $N$in $G$.
3. `Prove that every group of prime order is cyclic.
4. State and prove Cayley’s theorem.
5. Show that any two finite cyclic groups of the same order are isomorphic.
6. Define a subring of a ring. Show that the intersection of two subrings of a ring $R$ is a subring of $R$.
7. Show that every finite integral domain is a field.
8. Show that every Euclidean ring is a principal ideal domain.

**PART – C (2 × 20 = 40)**

**Answer any TWO questions**

1. (a) State and prove the fundamental theorem of arithmetic.

(b) Show that a nonempty subset $H$ of a group $G$ is a subgroup of $G$ if and only if $a, b\in H$implies $ab^{-1}\in H$.

1. (a) State and prove the Lagrange’s theorem.

(b) Let $R$ be a commutative ring with unit element whose only ideals are $\left(0\right)$ and $R$ itself. Show that $R$ is a field.

1. (a) Determine which of the following are even permutations:
2. $\left(\begin{matrix}1&2&3\\2&3&4\end{matrix}\begin{matrix}4&5&6\\5&1&6\end{matrix}\begin{matrix}7&8&9\\7&8&9\end{matrix}\right)$ (ii) $\left(1, 2, 3, 4, 5\right) (1, 2, 3)$

(b) If $G$ is a group, then show that $A(G)$, the set of automorphisms of $G$, is also a group.

1. (a) Show that an ideal of the Euclidean ring $R$ is a maximal ideal of $R$ if and only if it is generated by a prime element of $R$.

(b) Show that $Z(i)$, the set of all Gaussian integer, is a Euclidean ring.

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